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Spin Dependent Quark Forces and the Spin Content of the Nucleon

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The spin crisis of the nucleon is that the quark spin contribution is only a small fraction of the nucleon spin. A relativistic Dirac equation approach is followed assuming three low mass current quarks in the nucleon described by a $(1/2^+)^3$ configuration. If the lower component contribution to the normalization of the quark wave function is about 0.18, then the axial charge of the nucleon can be reproduced. However including the same lower component to every quark wave function is not enough to resolve the spin crisis. The net u quark spin z component is predicted as 1.0 and the net d quark spin z component is predicted as -0.25, both in disagreement with experiment. These predictions can be brought into agreement with experiment if flavor independent but spin dependent forces are assumed between the quarks. The strength of the spin dependent force found by empirically fitting the nucleon spin data is shown to be comparable to the spin dependence that can explain the Δ-nucleon mass difference. The spin content of the Δ^+ is then predicted using the interactions that reproduce the spin content of the proton.

KEY WORDS: nucleon spin content; quark forces; spin dependent forces; current mass quarks.

1. INTRODUCTION

Polarized projectiles such as electrons and muons, scattering from polarized targets such as protons, deuterons, Helium^3, and Lithium^6 have been used to experimentally determine (Hughes and Voss, 1999) the quark spin content of the nucleon. The Helium^3 was modeled as two protons with spins in opposing directions, and the neutron spin in the direction of the Helium^3 spin. The Lithium^6 was modeled as a polarized deuteron with a spinless alpha particle partner. The beta decay of various hyperons (Jaffe and Manohar, 1990) has led to a reduced expectation for the fraction of the proton spin carried by quarks. This is connected to the F and D invariant matrix elements of the axial current, whose sum is $|g_a/g_v| = F + D$, which experimentally (Close and Roberts, 1993) is about 1.25.

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This shows the "naïve" expectation for the fraction of the proton spin carried by quarks is not unity, but more like 0.75. Sehgal has suggested (Sehgal, 1974) that there is an orbital angular momentum contribution from quarks to the nucleon spin. This comes about naturally with a Dirac description of the $J^{\pi} = (1/2^{+})$ state. The lower component has orbital angular momentum unity, and 2/3 of the time the quark spin z component is oppositely directed to the z component of total angular momentum of the quark.

The proton is modeled as two u quarks and a d quark, in a $(1/2^{+})^{3}$ configuration coupled to total angular momentum 1/2, with $J_z = 1/2$. The flavor and J_z part of the three quark model of the proton wave function can be written as

$$|\Psi J_z = +1/2\rangle = (1/\sqrt{6})[2u\!\uparrow\! u\!\uparrow\! d\!\downarrow - u\!\uparrow\! u\!\downarrow\! d\!\uparrow - u\!\downarrow\! u\!\uparrow\! d\!\uparrow] \tag{1}$$

With a flavor and spin independent Dirac equation approach, and a $(1/2^{+})^{3}$ configuration of three quarks in a nucleon, [u, u, d for the proton] the$(1/2^{+})$ [$J = 1/2$, parity positive] wave function for such a spin up quark can be written, with q standing for u or d, as

$$q\!\uparrow = (1/r)\left[\ F Y_{00}\chi_{1/2,1/2}\right]$$
$$\left[\ i G \Sigma_{\mathrm{ml,ms}}\, C_{\mathrm{1ml,1/2ms,1/21/2}}\, Y_{\mathrm{1ml}}\ \chi_{\mathrm{1/2ms}}\right]. \tag{2}$$

Here C is a Clebsch-Gordan coefficient, ml and ms are the orbital and spin z components of angular momentum, and $\chi_{1/2\mathrm{ms}}$ is the quark spinor. The spherical harmonics are the Y_{LM}. F is the radial part of the upper component of the wave function which survives in the nonrelativistic limit. G is the radial part of the lower component quark wave function. The normalization of the one quark wave function is

$$1 = \int [F^2 + G^2]\, dr \tag{3}$$

A relativistic bag model (Golowich, 1975) has been used to reproduce the axial charge of the nucleon by adjusting the average u, d quark mass to about 0.12 GeV. The contribution to the axial charge of such a configuration is

$$|g_a/g_v| = (5/3)\int [F^2 - G^2/3]\, dr \tag{4}$$

If the lower component contribution to the norm, $N_1 = \int G^2 dr$, is about 0.187, then such a relativistic wave function can reproduce the axial charge of the nucleon.

Table I below shows how including such a lower component contribution to the normalization improves the comparison of theory to experiment. With this spin up proton wave function, the probability of finding a quark of a given flavor, and spin direction is shown, first assuming no lower component contribution to the normalization, and then assuming that the axial charge is fit by adjusting the lower component contribution to the normalization.

Table I. Predicted Quark Spin Content in the Nucleon Including a Lower Component to the Quark Wave Function

Flavor	$N_1 = 0$	$N_1 = 0.1875$	Experiment
$\langle u\uparrow\rangle$	5/3	5/4	
$\langle u\downarrow\rangle$	1/3	1/4	
Δu	4/3	1.0	0.77 ±0.10 ±0.08
$\langle d\uparrow\rangle$	1/3	1/4	
$\langle d\downarrow\rangle$	2/3	1/2	
Δd	−1/3	−1/4	−0.52 ±0.14 ±0.09
$\Delta u + \Delta d$	1	3/4	0.58 +/0.01

Δu is the number of u quarks with spin component up minus the number of u quarks with spin component down. Δd is the number of d quarks with spin up minus the number of d quarks with spin down. Assuming for each quark the same Dirac wave function whose lower component contribution to the normalization was chosen to fit the axial charge lessens, but does not remove, the disparity with experiment for Δu and Δd. Balitsky and Ji (1997) attribute the spin discrepancy to gluons. Spin dependent forces are here considered as a way to resolve this discrepancy between experiment and the quark contribution to the nucleon spin assuming the $(1/2^+)^3$ configuration is valid. These spin dependent forces can well be the result of one or multiple gluon exchange between quarks.

2. SPIN DEPENDENT FORCES

The pairwise interactions between quarks in the $\Delta^{3/2}$ particle occur solely in the $S = 1$ two quark spin state, while in the nucleon the quark–quark interaction is a combination of $S = 0$ and $S = 1$ two quark spin states. Therefore the $\Delta^{3/2}$-nucleon mass difference suggests spin dependent forces are present between two quarks, with $S = 1$ spin states having higher energy than $S = 0$ spin states. Consider now the quarks in the proton spin up state. Flavor invariance is still assumed for the quark–quark forces. A spin down quark interacts with each of the other two spin up quarks, 50% in a spin zero state, and 50% in a spin one state. It is somewhat different for a spin up quark, whose spin is parallel to the proton spin. When interacting with the other spin up quark, the spin state is 100% $S = 1$. When interacting with the spin down quark, it is 50% spin zero, and 50% spin one. So attributing the $\Delta^{3/2}$-nucleon mass difference to spin dependent forces, there is also a difference in the interactions for spin up and for spin down quarks in the nucleon with spin up. For a quark with spin up, 25% of the time it interacts with other quarks in an $S = 0$ spin state, and 75% in an $S = 1$ state. For the quark with spin down, 50% of the time it interacts with other quarks in an $S = 0$ spin state, and 50% of the time in an $S = 1$ state.

2.1. Determination of the Quark Spin Up, and Spin Down Normalization Relations

Define $P = \int[F^2 - G^2/3]dr$ for a spin up quark in the spin up proton, and also define $M == \int[F^2 - G^2/3]dr$ for a spin down quark in the spin up proton. This is valid for any flavor quark in the proton, assuming flavor invariant forces. Allowing for differences between the upper and lower component contributions to the normalization due to spin dependent two quark forces, one can fit the $|ga/gv|$ nucleon axial charge by asking P and M to satisfy

$$1.2601 = [4P + M]/3 \tag{5}$$

Also, assuming no strange quarks in the proton, the quark spin contribution to the spin up proton is Δu plus Δp, well determined experimentally (Hughes and Voss, 1999) as 0.58. With spin dependent quark forces, P and M now must also satisfy

$$0.58 = 2P - M. \tag{6}$$

The empirically determined values for P and M that fit the data are, $P = 0.7267$ and $M = 0.8734$. These values predict $\Delta u = [5P - M]/3 = 0.920$ and $\Delta d = [P - 2M]/3 = -0.340$. These predictions fall within the systematic and statistical error bars (Hughes and Voss, 1999) for the experimental nucleon values of 0.77 ± 0.10 ± 0.08 and -0.52 ± 0.14 ± 0.09 respectively. These values of P and M allow for the determination of the lower component contribution to the quark wave function. For spin up quarks, $N_1 = 0.205$, and for spin down quarks, $N_1 = 0.095$. These values deviate from the 0.1875 needed on an average to reproduce only the axial charge. The difference in the lower component contribution to the normalization is attributed to spin dependent quark forces.

An analytic one body Dirac equation model is used to systematize these ideas. The radial part of the Dirac equation for the $(1/2^+)$ state, with scalar and vector potentials S, and V is

$$[m + S - E + V]F + [-1/r - d/dr]G = 0$$
$$\text{and} \quad [-1/r + d/dr]F + [-m - S - E + V]G = 0. \tag{7}$$

A nearly exponential shape for the proton radial wave function can be inferred from nonrelativistic dipole fits to the proton charge form factor. To obtain an exponential radial wave function, consider $F = A\,r\exp[-Lr]$, and $G = B\,r^2\exp[-Lr]$ as the upper and lower component radial wave functions. A and B are determined by the normalization condition. The size parameter L is about 0.71 GeV from dipole fits to the charge form factor. With the scalar potential,

$$S = Lr(E - m)/6 - (E + m)/2 + 3L/2(E - m)r \tag{8}$$

and the vector potential,

$$V = Lr(E - m)/6 + (E + m)/2 - 3L/2(E - m)r, \tag{9}$$

these wave functions will satisfy the Dirac equation. For u or p current quark masses, the quark mass is small (Barnett *et al.*, 1996), so m is set to zero. This potential has a linear scalar confining term plus a constant and a coulombic attractive term. L and E are parameters that describe a linear confining potential and a coulombic attractive one body potential. This may be a remnant of a one gluon exchange between two quarks. However the quark dynamics are modeled here by a one body potential in the Dirac equation. This is like a shell model with independently moving quarks within the nucleon. The ratio of the lower to upper component contribution to the normalization is $N_1/N_0 = E^2/3L^2$. The scalar and vector potentials have been parameterized to depend on L and E, which is also the single quark energy eigenvalue. In this model, these parameters change for quark spins parallel or antiparallel to the nucleon spin. These parameter changes reflect the spin dependent interactions with the other quarks in the nucleon.

Fitting the nucleon spin data via spin dependent forces allows the determination of the lower and upper component contributions to the normalization. Using the exponential analytic model Dirac equation, for a quark spin parallel to the proton spin, this determines the parameter ratio, $0.205/0.795 = E^2/3L^2$. This spin parallel quark undergoes interactions with the other quarks that are 75% $S = 1$, and 25% $S = 0$. For a quark with spin antiparallel to the proton spin, the spin content data are fit if the potential parameters satisfy $0.095/0.905 = E^2/3L^2$. In a proton with spin up, a quark with spin antiparallel to the proton spin undergoes interactions that are 50% $S = 1$, and 50% $S = 0$. These numbers correlate to $E/L = 0.8794$ for spin up, and $E/L = 0.5609$ for spin down quarks in a proton with spin up. These suggest for the Δ particle, with 100% $S = 1$ interactions, that $E/L = 1.10$. Further, this correlation suggests, for the spin up proton, containing two quarks with spin up, and one with spin down, that the proton energy is proportional to $2(0.8794) + 0.5609 = 2.3197$. The Δ particle, with three quarks undergoing 100% spin 1 interactions, would have an energy proportional to $3(1.10) = 3.30$. Thus this correlation suggests the proton/Δ mass ratio to be 0.696 versus 0.759 experimentally. If the size parameter L, is set to 0.71 GeV as suggested by dipole fits to the proton magnetic moment, then the proton rest frame energy is estimated as 0.988 GeV and the Δ rest frame energy is 1.41 GeV. These numbers have allowed for some elimination (Strobel, 2001) of the center of mass energy, when going from an independent motion of three quarks frame work into the rest frame where the center of mass is fixed. With a linear scalar potential, and massless quarks, the relation used is $E^2_{\text{rest}} = 0.6\,E^2_{\text{independent motion}}$. These calculated nucleon and Δ particle energies show that the Dirac equation with a linear confining potential and spin dependent parameters determined by an empirical fit to the nucleon spin content data compare well with experiment. Thus the strength of the spin dependent potential found fitting the spin content of the nucleon well matches the spin dependence that can explain the Δ nucleon mass difference.

Table II. Quark Spin Content of the Δ^+ for Various J_z Values

	J_z	
Flavor	1/2	3/2
$\langle u\uparrow\rangle$	0.815	1.223
$\langle u\downarrow\rangle$	0.407	0
Δu	0.408	1.223
$\langle d\uparrow\rangle$	0.408	0.611
$\langle d\downarrow\rangle$	0.204	0
Δd	0.204	0.611
$\Delta u + \Delta d$	0.612	1.834

2.2. Prediction of the Δ^+ Spin Content

The exponential analytic spin dependent independent particle model allows prediction for the spin content of the Δ^+, and other charge states as well. The singly positive charged state is considered here, with z component of J equals $1/2$ or $3/2$. The lower component of the quark wave function is taken as having a normalization of 0.2915. Assuming spin dependent forces between the quarks, this value is inferred from the proton analysis of quark spin parallel and antiparallel to the nucleon spin. The predicted Δ^+ spin content for various possible J_z values is shown in Table II.

3. CONCLUSIONS

A flavor independent, spin dependent, quark–quark interaction is suggested to explain the spin crisis of the proton. For the spin up proton, with u, u, p quarks, there are two quarks with spin up, and one quark with spin down. If the quark with spin down has a smaller lower component contribution to the normalization than does a spin up quark, then the experimental values of $|ga/gv|$, and the spin contribution to the proton can be reproduced. Fitting these numbers determine the lower component contribution to the normalization is 0.095 and 0.205 for the spin down and up quarks respectively. Then Δu is predicted to be 0.920 and Δd is predicted to be -0.340, both numbers within the experimental error bars. The lower component contribution to the normalization is found to be 0.205 for a quark with spin parallel to the proton spin, and 0.095 for a quark with spin antiparallel to the proton spin. This compares to an averaged value of 0.1875 for the lower component normalization needed to reproduce the proton axial charge. Such differences in the lower component contribution to the normalization can come from spin–spin forces in two body quark interactions. A simple analytical exponential wave function model shows that the magnitude of the $S = 1$ and $S = 0$

spin state interactions is comparable to that needed to explain the Δ-nucleon mass difference. In a proton with spin up, a spin up quark sees a different mix of spin states than does a spin down quark. For a quark with spin up, 25% of the time it interacts with other quarks in an $S = 0$ spin state, and 75% in an $S = 1$ state. For a quark with spin down, 50% of the time it interacts with other quarks in an $S = 0$ spin state, and 50% of the time in an $S = 1$ state. The exponential model of a quark Dirac radial wave function requires the $S = 1$ interaction to be higher energy than the $S = 0$ interaction in explaining the empirical quark spin content in the proton. This energy difference is comparable to what is needed to explain the Δ-nucleon mass difference. Using the exponential one body Dirac equation model, these spin–spin interactions are used to predict the spin content of the Δ^+ particle with $J_z = 1/2$ or $3/2$.

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